Advice for the Final

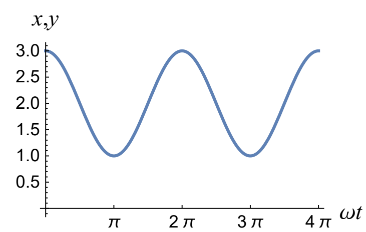
These notes are not intended to teach any new material, but instead provide advice on several concepts students typically struggle with and some general test taking procedures.

How to Rock Tests

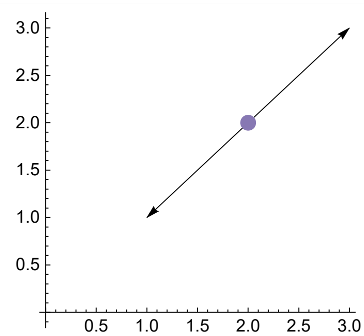
The following advice will help make your tests more legible. Psychologically speaking, the more time the graders need to spend trying to understand your work, the more frustrated they become and the fewer points they want to give you.

- Start every new question on a clean page
  - **Box your answers**
  - Clearly indicate the question numbers and all separate parts (i.e. Part a, Part b...)
  - Write the parts sequentially on your answer sheet (even if you don't work on them sequentially). If you force the grader's to hunt for your answers, they will not be happy
- Always show your work. No work = no partial credit
- Read problems carefully and answer all parts of a question exactly. If you are asked for the period of an oscillator, don't give ω, give $T = \frac{2\pi}{\omega}$
- Use the notation given in a problem
  - All of your answers must only consist of the variables given to you in the problem statement. For example, if you are asked for the oscillations of a mass m on a horizontal spring k starting from the fully extended position a distance A from the spring's equilibrium, do not write $A\,\mathrm{Cos}[\omega\, t + \theta]$, but instead write $A\,\mathrm{Cos}\left[\left(\frac{k}{m}\right)^{1/2} t\right]$
  - Don't change the names of variables given in the problem. If they call the initial velocity in the y-direction v_0, then don't call it v_y.
- Write something for every part of the problem. You might get partial credit
  - Even if you are stuck on Part (a) of a problem, look at the remaining parts. There are often easy parts that don't build on previous results
  - Do not write an entire paragraph of information phishing for partial credit. But do write down 1-2 formulas that you suspect are relevant. They may trigger a thought process that enables you to solve the problem
- **Extremely Important**: Check the units on all of your answers. If the question asks for a time, do not turn in a length. Common mistakes:
  - $\mathrm{Cos}\left[\frac{k}{m} t\right]$, you incorrectly remembered $\omega_{\text{spring}} = \frac{k}{m}$ (forgetting the square root) and wrote $\mathrm{Cos}[\omega\, t]$, but the units of $\frac{k}{m}$ are $\frac{1}{s^2}$ and cosine must always act on dimensionless quantities
  - $\frac{m+1}{k}$, you cannot add a quantity with dimensions (like a mass m) to a non-dimensional number like 1
  - $F = 9.8\, m$, please do not ever write $g = 9.8$, which not only unnecessarily clutters your formulas but also has the wrong units. Leave it as g, and if the question asks for a numerical answer, then only replace g by $9.8\ \frac{m}{s^2}$ in the very last step

    - Please note that when grading final exams, the TAs are extremely forgiving about every possible type of mistake *except incorrect units*! Ignore units at your own peril...
- Perform quick checks to make sure your answers make sense
    - It is often easy to find sign errors. For example, suppose you are asked to find the maximum height of an object that starts at a height z_0 with a speed v_0 pointing straight up. You carry out the algebra and find $z_0 - \frac{v_0^2}{2g}$. Does the minus sign make sense (i.e. if you start off going faster, shouldn't you reach an even greater height?)
- If you are asked to describe something, do so completely. For example, if you are asked to describe the motion of an oscillator given by $x = 2 + \text{Cos}[\omega t]$, $y = 2 + \text{Cos}[\omega t]$:
    - *Do not* write "the particle travels in a line"
    - *Do not* simply draw a graph with no description such as



    - *Do* write a complete description of this line such as "the particle travels in a line that lies along $y = x$"
    - *Or even better,* write "the particle travels in a line that lies along $y = x$ from (1, 1) to (3, 3)"
    - *Do* include a useful graph to show this motion



- Simplify your answers when possible
    - Turning in $z + \frac{v^2}{g} - \frac{v^2}{2g}$ instead of $z + \frac{v^2}{2g}$ smells of laziness. The graders are looking for simplified forms, so you may accidentally get points taken off if you do not simplify
    - More complex relations such as $\text{Sin}\left[\text{Cos}^{-1}\left[\frac{m g}{k x}\right]\right]$ are usually acceptable, but you should learn the trick to simplify this down to $\frac{(k^2 x^2 - m^2 g^2)^{1/2}}{k x}$, since in many problems this allows you to further simplify the result and get a really nice result. (Hint: The trick is to consider a right triangle with one side of length $m g$ and the hypotenuse of length $k x$. The angle between these two legs of the triangle will be, by definition, $\text{Cos}^{-1}\left[\frac{m g}{k x}\right]$, which allows you to take its Sine)
- Time permitting, double check your work. There are typically multiple ways to check your answers. For example, you could compute a cross product in two ways, and they both better give you the same result (this is how you check yourself for algebra mistakes!)

- Given $\vec{A} = A_x\,\hat{x} + A_y\,\hat{y} + A_z\,\hat{z}$ and $\vec{B} = B_x\,\hat{x} + B_y\,\hat{y} + B_z\,\hat{z}$, then $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$
- $\left|\vec{A}\times\vec{B}\right| = |\vec{A}|\,|\vec{B}|\,\text{Sin}[\theta]$ where θ is the angle between $\vec{A}$ and $\vec{B}$

Concepts Worth Reviewing

Ph 1a

- Formulas such as $d = \frac{1}{2}\,a\,t^2 + v\,t$ only apply when acceleration and velocity are constant. Don't use them when this assumption does not hold!
- Any equation must either have a scalar on both sides or a vector on both sides. Never write a mixed equation such as $\vec{L} = m\,r\,v$. Some more notes on vectors:
  - Vectors either a symbol (either $\rightharpoonup$ or ^) above them (Ex: $\vec{A}$, $\hat{y}$). Denote *all* of your vectors with one of these two symbols. Anything else is assumed to be a scalar. The only exception you should ever see (or write) is the zero vector, which is often written as 0 instead of $\vec{0}$ out of laziness (but it really is a vector)
  - If a question asks for a vector, and your answer is a scalar, you will not get full credit
  - You cannot multiply two vectors together. $\vec{A}\,\vec{B}$ does not make sense
  - The following operations yield a vector:
    - $\vec{A} + \vec{B}$
    - $\vec{A} - \vec{B}$
    - $\vec{A}\times\vec{B}$
  - The following operation yields a scalar:
    - $\vec{A}\cdot\vec{B}$
  - When $\vec{A} = \vec{B}$, the above operations can all be simplified
    - $\vec{A} + \vec{A} = 2\,\vec{A}$
    - $\vec{A} - \vec{A} = \vec{0}$
    - $\vec{A}\times\vec{A} = \vec{0}$
    - $\vec{A}\cdot\vec{A} = |\vec{A}|^2 \equiv A^2$
- For circular motion in the x-y plane, the corresponding $\vec{\omega}$ and $\vec{L}$ vectors point out of the page in the $\hat{z}$ direction. This direction is given by the right-hand rule
- $\vec{\tau} = \vec{r}\times\vec{F}$ and (for a point particle) $\vec{L} = m\,\vec{r}\times\vec{v}$. This is a *cross product*, not a *dot product*. Therefore, *the order matters* because the cross product is anti-symmetric: $\vec{r}\times\vec{F} = -\vec{F}\times\vec{r}$
- The mnemonic for the cross product is only useful if you remember how to compute matrix determinants (there are tricky minus signs here!) so go through these two calculations and make sure that you understand the sign of each term.

- Given $\vec{r} = a\,\hat{x} + b\,\hat{y}$ and $\vec{F} = k(a\,\hat{x} + b\,\hat{y})$, the torque equals $\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ a & b & 0 \\ k\,a & k\,b & 0 \end{vmatrix} = (k\,a\,b - k\,a\,b)\,\hat{z} = \vec{0}$. *This minus sign is **critical***, so make sure you understand it! Of course, since $\vec{r}$ and $\vec{F}$ are parallel, the angle between them is 0 so that $|\vec{\tau}| = |\vec{r}|\,|\vec{F}|\,\text{Sin}[\theta] = \vec{0}$
- Given $\vec{A} = A_x\,\hat{x} + A_y\,\hat{y} + A_z\,\hat{z}$ and $\vec{B} = B_x\,\hat{x} + B_y\,\hat{y} + B_z\,\hat{z}$, then $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y\,B_z - A_z\,B_y)\,\hat{x} + (A_z\,B_x - A_x\,B_z)\,\hat{y} + (A_x\,B_y - A_y\,B_x)\,\hat{z}$

Ph 1b

The following notes will become useful in Ph 1b

- Vectors have a magnitude and direction. $\vec{E} = \frac{k\,q}{r^2}$ should make you cringe - add the direction $\vec{E} = \frac{k\,q}{r^2}\,\hat{r}$ within the equation
- Check your units. If you are computing $\vec{\nabla}\cdot\vec{E}$ and you come up with an answer $\frac{\rho}{a\,b\,L}$ (where ρ is a charge density and a, b, L are lengths), you know something went horribly wrong since $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$. So you definitely dropped the ϵ_0 in the denominator, and you somehow acquired an incorrect meters^3 in the denominator
- The potential $\phi = -\int_\infty^r \vec{E}\cdot d\vec{s}$ is a line integral from infinity (the reference point with zero potential)
- For volume integrals, use the appropriate formula for the infinitesimal volume element depending on your coordinate system:

$$dV = \begin{cases} dx\,dy\,dz & \text{Cartesian} \\ r^2\,\text{Sin}[\theta]\,dr\,d\theta\,d\phi & \text{spherical} \\ r\,dr\,d\theta\,dz & \text{cylindrical} \end{cases} \tag{1}$$

- Charge does not flow across capacitors. Rather, charge goes around the circuit and a charge difference slowly accumulates between the two plates until the current vanishes